Problem Set 5
Dual Simplex and Integer Programming

AM121/ES121 — Fall 2019

Due 5:00 PM, Thursday, October 31, 2019

Announcements

- The assignment is due by 5:00 PM, Thursday, October 31, 2019.
- You may work with another student on this assignment and submit just one writeup. **But you must work together on every problem and state that you did this on your submission.** It is ok to divide the writing up of the solutions, but not solving the problems. It is ok to choose to complete the assignment alone too. The workload should be manageable.
- Be sure to record both your names in the submission.
- Readings: Bradley, Hax, and Magnanti, Section 3.9 and 4.9; Jensen and Bard, Chapter 7.

Goals

In this assignment you will apply your knowledge of integer programming and modeling to a variety of problems.

Contents

1 Resolve Moo.

You offered consulting services to *Mike's Milkin' Cattle Ranch* last time where you have found the optimal tableau for the following LP, with s_1, s_2 and s_3 being the slack variables.

$$\begin{aligned} \text{maximize} \quad & 2.5x_1 + 2x_2 + 3x_3 \\ \text{subject to:} \quad & x_1 + x_2 + 2x_3 + s_0 = 120 \\ & x_1 + s_1 = 50 \\ & x_2 + s_2 = 50 \\ & x_3 + s_3 = 50 \\ & x_1, x_2, x_3, s_0, s_1, s_2, s_3 \geq 0 \end{aligned}$$

The optimal tableau is as follows

$$\begin{bmatrix} & z & x_1 & x_2 & x_3 & s_0 & s_1 & s_2 & s_3 & rhs \\ z & 1 & 0 & 0 & 0 & 1.5 & 1 & 0.5 & 0 & 255 \\ x_1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 50 \\ x_2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ x_3 & 0 & 0 & 0 & 1 & 0.5 & -0.5 & -0.5 & 0 & 10 \\ s_3 & 0 & 0 & 0 & 0 & -0.5 & 0.5 & 0.5 & 1 & 40 \end{bmatrix}$$

While Mike thanks you for your hard work, he remembers that he has forgotten to include a production constraint: $x_2 + x_3 \geq 70$. "*What do we do?*"

Task 1

1. With this new constraint, is the current optimal solution still feasible?
2. Instead of re-solving the problem from scratch, add this constraint (as an equality constraint with its associating slack variable) to the optimal tableau. Bring the tableau into dual-feasible form, and perform one or more dual-simplex pivots to solve the problem with the new constraint. What's the optimal solution and the corresponding profit?

End Task 1

2 Quark is back.

After a visit from two Nausicaans, Quark decided that he better produce whole units of video game systems after all.

Task 2

Modify the AMPL model file from assignment 1 (`prod.mod`) to take integrality into consideration. Re-solve with the data file (`console.dat`), using CPLEX as your solver. What is the optimal production levels and expected profit now? Compare the solution to your solution from assignment 1, and note any interesting similarities or differences.

End Task 2

3 How we roll

A new bakery chain called 'How we roll' has hired you to manage their advertisement campaign. The founder, Ando, is considering advertisement options in print, online, and television, and has provided you with the following options:

Prints	New York Times	9th Wonder	Las Vegas Today	
Costs	9	5	11	
Expected returns	13	8	13	
Online	**Sponsored Search**	**AdWords**	**NBC.com**	**SuperHiro.org**
Costs	12	7	11	2
Expected returns	27	13	13	4
Television	**NBC daytime**	**NBC primetime**	**NBC late night**	
Costs	30	60	40	
Expected returns	40	80	50	

Notice that each option can only be chosen once, but multiple options may be selected within a particular advertising avenue. All costs and expected returns are in thousands of dollars. While Ando wants to go big, the company's ad budget is limited to 100 thousand dollars. How can 'How we roll' choose among its advertising options to maximize returns (thinking of our budget as a sunk cost)?

Task 3

1. Formulate a mathematical model of the problem as a mixed integer program. Be sure to specify the type (binary, integer, or continuous) of each variable and describe the elements of your model.
2. Implement your program in AMPL. Ensure that your model is general enough to allow for addition of advertising avenues (i.e. radio) or options (i.e. another magazine), and expressive enough to allow for the kind of things we are asking you to do below. In a couple of sentences, describe the elements of your AMPL program in as much as they do not follow immediately from the mathematical model. Be sure to explain what kind of things can be changed that will provide for the flexibility your formulation will need.

   Word to the wise: work to use sets and variables and parameters defined on these sets as much as possible, or be prepared to do more work later.
3. What is the optimal ad campaign? How much would we generate in expected returns?
4. Write down the *linear* constraint necessary to meet each of the following conditions:
   - Include at least one TV option in the campaign.
   - Advertise on Adwords or Sponsored Search (choose at least 1) but not both.
   - If advertise on 9th Wonder, then advertise on SuperHiro.org.
   - Either advertise on NBC.com and NBC primetime, or don't advertise on either.
5. Ando believes that meeting these additional conditions will benefit the ad campaign. Give a reason against Ando's statement, along with the assumption that needs to be met to support your reasoning. Then give a reason in favor of Ando's statement, and the assumption that needs to be met for it to be true.
6. Formulate a model with all the additional constraints in AMPL and solve it. How exactly should we roll? Do we make good money? (**Note:** If you are trying to index into a specific ad by its name in the model file, you must put the name in double quotes. No changes need to be made in the data file.)

End Task 3

4 Zipcars

We wish to set up two ZipCar lots in the river side of the Harvard residential district. We aim mostly to serve the communities living in Peabody Terrace, 20 Mount Auburn, 18 Banks Street, 5 Cowperthwaite Street, and 8 Dewolfe Street. We are considering three possible locations (of which we would open two lots): 27 Banks Street, 10 Grant Street, and 15 Mt. Auburn Street. We wish to minimize the total distance our customers in these locations would have to travel to reach a lot. (i.e. calculate the "distance to nearest lot" for each community and then minimize this sum) All the data is provided for you.

Task 4

1. Take a look at the provided data file `zipcar.dat`. Write down a mathematical model as an integer program that captures the constraints of the problem. Describe the elements of your model. For now, ignore the population data and lot supply from `zipcar.pop.dat`.
2. Implement the model in AMPL and solve it with the provided data file `zipcar.dat`. What do you find?
3. Take a look at the provided data file `zipcar.pop.dat`. Modify your mathematical model to consider the supply constraints at the lots and the user population in each of the communities. For the objective, aim to minimize the total distance traveled by all customers. Describe the (modified) elements of your model. (Note: a community's population may split up and go to different lots)
4. Implement the model in AMPL and solve it with the provided data file `zipcar.pop.dat`. What do you find?

End Task 4

5 Candidates, meet the press

During the 2016 presidential elections, the National Press scheduled a one-day 'Meet the candidates' event, where four reporters held interview and debate sessions with the candidates. They were:

- Rita Skeeter, from *The Daily Prophet*.
- Will Thacker, from *Horse and Hound*.
- Andy Sacks, from *Runway*.
- Clark Kent, from *Daily Planet*.

and their schedules:[1]

	10–11	11–noon	1–2	2–3	3–4
Rita	✓	✓	✓	Debate	Debate
Will	×	×	✓	✓	✓
Andy	✓	✓	✓	Debate	×
Clark	Debate	✓	×	✓	✓

[1] Easter Egg: Rita is obviously available all day. Will had no idea he was even going to be at a press event. Andy had to run off early because Miranda Priestly has to. Clark is unavailable from noon to 2 because that's when all the accidents happen.

where a ✓ indicates that the reporter is available to meet for 1 on 1 interviews at that time, and 'Debate' indicates that the reporter is willing to host a 2 candidate debate (but can hold interviews in those slots as well). The candidates participating in the event were:

Democrats	Republicans
Former Sec of State Hillary Clinton (Leading)	Businessman Donald Trump (Leading)
Senator Tim Kaine (Leading)	Governor Mike Pence (Leading)
Senator Bernie Sanders	Governor John Kasich
Senator Jim Webb	Governor Chris Christie
Senator Elizabeth Warren	Former CEO Carly Fiorina
Harvard Law Professor Lawrence Lessig	Senator Ted Cruz
Governor Lincoln Chafee	Senator Marco Rubio
Governor Martin O'Malley	

And their schedules:

	10–11	11–noon	1–2	2–3	3–4
Clinton	×	✓	×	✓	✓
Kaine	✓	×	✓	✓	✓
Sanders	×	✓	✓	×	×
Webb	✓	×	×	✓	✓
Warren	×	✓	×	×	✓
Lessig	✓	✓	×	×	×
Chafee	✓	✓	✓	✓	✓
O'Malley	×	×	✓	×	×
Trump	✓	×	✓	×	×
Pence	×	✓	×	✓	×
Kasich	✓	✓	×	✓	✓
Christie	×	×	✓	✓	✓
Fiorina	×	×	×	✓	✓
Cruz	✓	✓	×	✓	✓
Rubio	✓	✓	✓	×	×

The National Press would like the candidates to participate in as many sessions as possible, as long as each candidate is present in at least one session.

Task 5

1. Provide a mathematical formulation of an integer program to schedule the interviews and debates so as to maximize the total number of candidate appearances. Describe the elements of your formulation. Then, formulate the model in AMPL and solve. In entering the data, you may find the provided file `candidates.dat` to be a useful starting point.

   How many candidate appearances can you schedule in? What is the schedule? Comment on any issues you see with the schedule.

2. Extend your mathematical formulation to include as a constraint that no candidate will appear on a particular reporter's show more than once (to avoid accusations of favoritism). Add the constraint to your AMPL code and re-solve. What's the schedule now?

3. In addition, the National Press would like to have the leading candidates appear on at least two different programs. Extend your mathematical formulation appropriately and mirror the change in AMPL. What's the schedule now?

End Task 5

6 Turning in your assignment

Final Task 6

Turn in your assignment to Gradescope by 5:00 PM, Thursday, October 31, 2019. Scan your homework if you completed it on paper and make sure the PDF is eligible. For AMPL exercises, this includes any model and data files you have created. When writing down the solution from AMPL, always include both the objective value and the values assigned to variables (when the program is feasible and bounded.) Scan the AMPL model and data files you have created for this assignment, and compile them into a single PDF for submission.

End Task 6

Congratulations on completing your fifth AM/ES 121 assignment!