AM/ES 121: Intro to Optimization Models and Methods

Fall 2018

Lecture 1: Course Overview

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SEAS

Meeting Times

- Mondays and Wednesdays 9-10:15
- No more Harvard time!

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Lesson Plan

- Review syllabus, schedule
  - Course website: am121.seas.harvard.edu
  - Piazza for Q&A
  - Canvas for submitting psets

- Overview of topics (with motivating examples)
  - Linear Programming
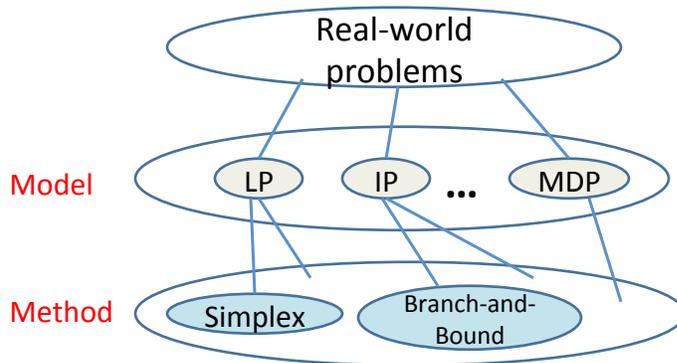 - Integer Programming
  - Stochastic Programming

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Mathematical Programming

- Set of methods to model and solve optimization problems with continuous or integer variables

- Programming ≠ computer programming
- Programming = Planning

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Mathematical Programming

- We emphasize modeling
- Methods with elegant mathematics
- Applications

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Prominent Applications

- Optimal crew scheduling saves American airlines >20 million dollars per year
- Optimization and analytics tools save UPS 85 million miles each year, saving around $125 m (how many routes can a driver with 25 packages choose from?) (and…)
- Amazon wouldn't exist without optimization
- Google PageRank– a tool for indexing the Web –is a calculation on a Markov chain.
- Clearing Nationwide Kidney Exchanges, saving lives.

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Campaign stops: The problem of the travelling politician
By WILLIAM COOK (Georgia Tech)

- What's the [quickest way](#) to visit all 99 counties in Iowa?
- Republican primary, 2011-12. Presidential caucus in Iowa.
- Swing state (but Democratic since 2008).

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# Solution (the fastest politician!)

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The Shortest Possible Baseball Roadtrip

- Ben Blatt (class of 2013)

  - http://www.wsj.com/articles/ SB100014240527023036574045763 57560436672964
  - https:// harvardsportsanalysis.wordpress.co m/2011/06/03/roadtrip/

  Ben, a sports analytics wizard, loves baseball. Eric, his best friend, hates it. But when Ben writes an algorithm for the optimal baseball road trip—an impossible dream of seeing every pitch of 30 games in 30 stadiums in 30 days—who will he call on to take shifts behind the wheel, especially when those shifts include nineteen hours straight from Phoenix to Kansas City? Eric, of course. Will Eric regret it? Most definitely.
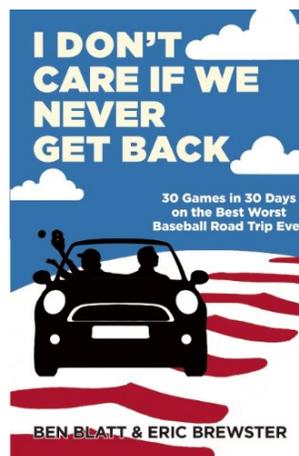
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Syllabus

- **Prerequisite**: AM 21b or Math 21b (linear algebra)

- **Recommended Text**: "Operations Research: Models and Methods", by Jensen and Bard.
- **Optional text**: "AMPL", by Fourer, Gay and Kernighan

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Syllabus

- 7 problem sets (mixture of modeling, computation, and theory). Encourage use of Matlab/Mathematica
  – late days
- Two "Extreme optimization" team assignments
  – what's that?
  (Extreme values: *communication, simplicity, feedback, courage, respect*).
- Two in-class midterms; No final.
- Grade breakdown is 30 % problem sets, 20% extreme optimization, 50% midterms

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Syllabus

- We'll be using **AMPL** and **CPLEX**

- AMPL is a **modeling tool**. Literally "A Mathematical Programming Language" (AMPL)
- Very simple way to write down IPs and LPs
- Set this up and test this week!

- CPLEX is a **solver**. Was developed by ILOG, now owned by IBM. Industrial strength.

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Syllabus

- Collaboration policy
  - Can collaborate in planning and thinking through solutions, but <u>must </u>write up your own solutions without checking this over with another student.
  - Do not pass solutions to problem sets nor accept them from another student. (It's NOT OK to get solutions to previous years' problem sets.)
- Engineering students, particularly S.B. concentrators, <u>should enroll under ES 121</u>. This will ensure the course counts as an elective.

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Get Ready for the Course

- Bookmark the course website http://am121.seas.harvard.edu/
- Complete Assignment 0 by Monday 9/10
  - Sign up for Piazza (bookmark it!)
  - Install AMPL
  - Complete an online sectioning form (by Tuesday 9/11)
- Check out the "Course logistics overview" document on the Resources page of the course website.
- My office hours: Wednesdays 10:20-11:30, MD 233
- **Attend sections!**
- Piazza (all announcements, all Q&A)

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Rough Schedule

- 6 weeks on **linear programming**
- 4 weeks on **integer programming**
- 2 weeks **stochastic programming**
- Combined with **applications**:
  - 2 extreme optimization projects (radiation treatment optimization, kidney exchange clearing)
  - 1 guest lecture on an application in LP/IP
- 2 **breakout modeling** sessions
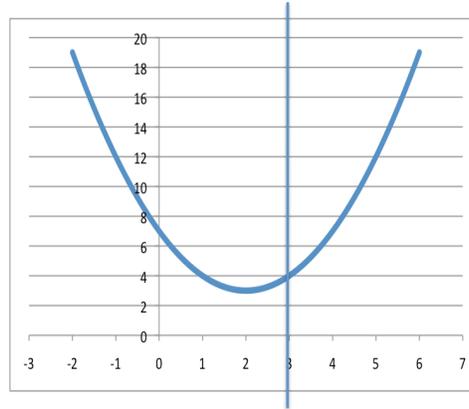
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Optimization

- The problem of making decisions to maximize or minimize an objective, maybe in the presence of complicating constraints
  - Decision variables
  - Objective function
  - Constraints

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A Simple Optimization Problem

- minimize $y = (x-2)^2 + 3$

minimize $y = (x-2)^2 + 3$
s.t. $x \geq 3$

This is non-linear. In this course, we focus on linear models.

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A Linear Program

$$\min \ z = 50x_1 + 100x_2$$
$$\text{s.t.} \quad 7x_1 + 2x_2 \geq 28$$
$$2x_1 + 12x_2 \geq 24$$
$$x_1, x_2 \geq 0$$

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Solving (graphically!)

$$\min \; z = 50x_1 + 100x_2$$
$$\text{s.t.} \quad 7x_1 + 2x_2 \geq 28$$
$$2x_1 + 12x_2 \geq 24$$
$$x_1, x_2 \geq 0$$

$$5 * (1) : 35x_1 + 10x_2 \geq 140$$
$$7.5 * (2) : 15x_1 + 90x_2 \geq 180$$
$$+ \; : 50x_1 + 100x_2 \geq 320$$

x_1=3.6, x_2=1.4, value 320
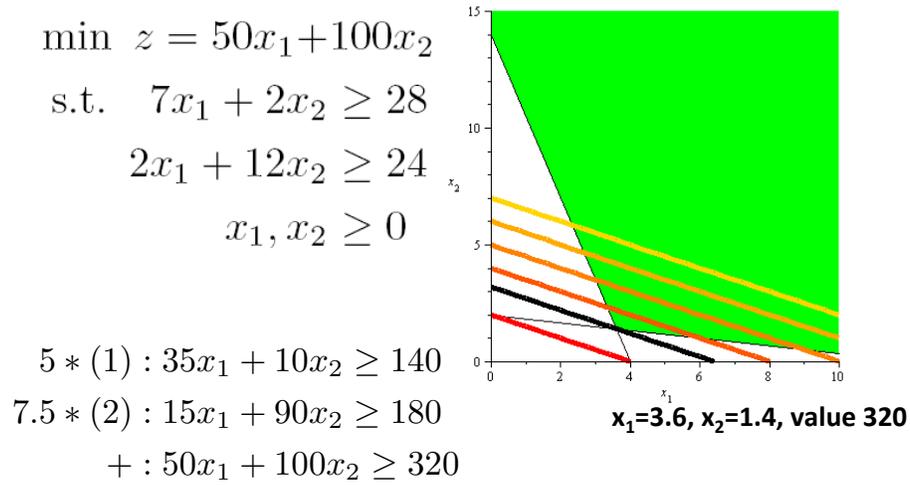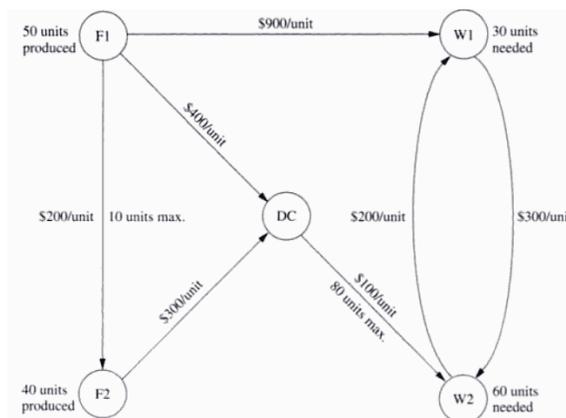
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A Distribution Network

Ship goods from factories to warehouses to minimize total cost while meeting demands and capacity constraints.

50 units produced F1 — $900/unit → W1 30 units needed

$400/unit

$200/unit 10 units max. DC $200/unit $300/unit

$300/unit

$100/unit 80 units max.

40 units produced F2

W2 60 units needed

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A Distribution Network

- Decision vars.: amount to ship on each lane,
  $x_{F1\text{-}F2}$, $x_{F1\text{-}DC}$, $x_{F1\text{-}W1}$, $x_{F2\text{-}DC}$, $x_{DC\text{-}W2}$, $x_{W1\text{-}W2}$, $x_{W2\text{-}W1}$.
- Objective: *minimize total shipping cost*
- Constraints:
  - *amount shipped in – amount shipped out = required amount*
  - *amount on a lane <= capacity*
  - *amount on a lane >= 0*

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Optimal Animal Feed!

- Produce the feed mix that satisfies nutrition requirements with minimal cost
- Requirements:
  - Calcium: at least 0.8% but not more than 1.2%
  - Protein: at least 22%
  - Fiber: at most 5%
- Nutrient Contents and Costs of Ingredients

Ingredient	Calcium	Protein	Fiber	Unit cost (cents/kg)
Limestone	0.38	0.0	0.0	10.0
Corn	0.001	0.09	0.02	30.5
Soybean meal	0.002	0.5	0.08	90.0

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What we'll cover for Linear Programming

- Modeling: Given a problem, formulate the linear programming mathematical model.

- Solve the formulated LPs
  - Simplex method
  - Sensitivity analysis
  - Duality

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Integer Programming (IP)

- Decision variables can only take **integer values**
- Linear programming model + integrality constraints
- Example:

$$\max \quad 5x_1 + 8x_2$$
$$\text{s.t.} \quad x_1 + x_2 \leq 6$$
$$5x_1 + 9x_2 \leq 45$$
$$x_1, \quad x_2 \geq 0, \textbf{\color{red}{integer}}$$

also "mixed integer programs" with {0,1}, integer and continuous

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Linear Programs vs. Integer Programs

- Linear Programs
  - Continuous decision variables
- Integer Programs
  - Discrete decision variables, must take integral values
  - Smaller feasible space

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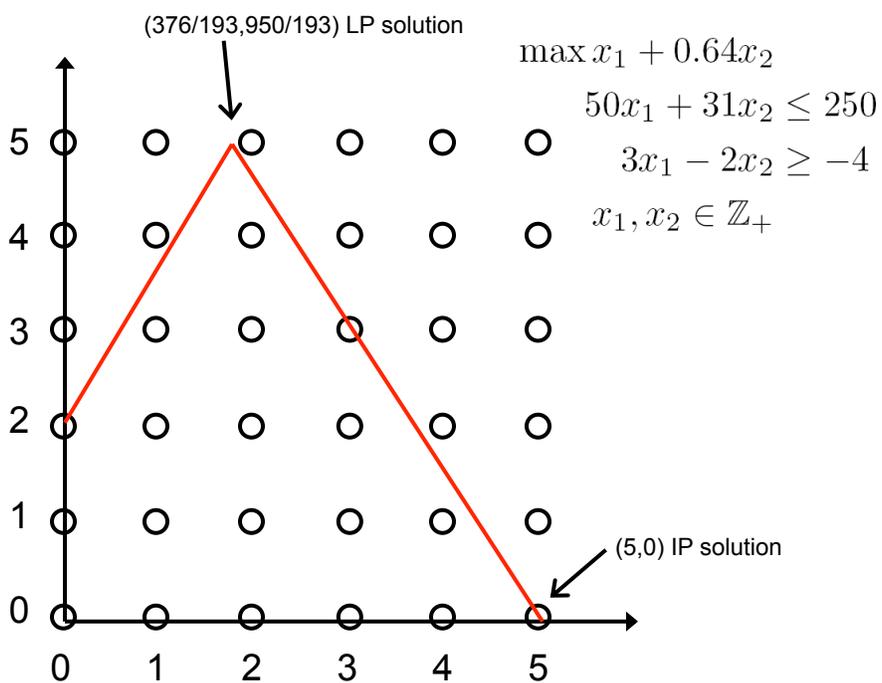
Linear Programs vs. Integer Programs

- Linear Programs
  - Continuous decision variables
- Integer Programs
  - Discrete decision variables, must take integral values
  - Smaller feasible space
  - Generally harder to solve!
- The natural idea of rounding LP solution for IP does not work...

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(376/193,950/193) LP solution

$$\max x_1 + 0.64x_2$$
$$50x_1 + 31x_2 \leq 250$$
$$3x_1 - 2x_2 \geq -4$$
$$x_1, x_2 \in \mathbb{Z}_+$$

(5,0) IP solution

Taking the fun out of Sudoku

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

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Taking the fun out of Sudoku

- Decision variables
  - $b_{i,j}^v$: equals 1 if cell at row *i* and column *j* has value *v*, otherwise 0.
- Constraints:
  - Each row must have one 1 ... 9
  - Each column must have one 1 ... 9
  - Each 3-by-3 section must have one 1 ... 9
  - Each cell must have a value from 1 ... 9
- Objective
  - Anything (it's a "constraint satisfaction problem", but can be solved via an IP by adopting any objective.)

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The Assignment Problem

- There are n people available to carry out n jobs
- Each person has to be assigned to exactly one job
- Person i if assigned to job j incurs cost c_{ij}
- Goal: find a minimum cost assignment

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What we'll cover for Integer Programming

- Modeling: Given a problem, formulate the integer programming mathematical model.

- Solve the formulated IPs
  - Branch-and-Bound
  - Cutting plane
  - Branch-and-cut

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Introducing Uncertainty

- So far, we've been thinking about using math. programming to solve problems with known model parameters.
- But many real world problems involve uncertainty.
- **Stochastic programming** is an approach for solving optimization problems that involve uncertainty.

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What we'll cover for Stochastic Programming

- Markov chains (to model an underlying stochastic process)
- Markov Decision Processes (MDPs), which we'll solve via linear programming
- Scaling up via two-stage, stochastic optimization

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Markov Chains

- A stochastic process (a sequence of random variables) that has the *Markov property*
- *Markov property*: state at time *t+1* only depends on state at time *t* (history before *t* does not matter)

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Markov Chains

- A stochastic process (a sequence of random variables) that has the *Markov property*
- *Markov property*: state at time *t+1* only depends on state at time *t* (history before *t* does not matter)
- Eg. Machine condition

State	Condition
0	Good as new
1	Minor deterioration
2	Major deterioration
3	Inoperable

State	0	1	2	3
0	0	7/8	1/16	1/16
1	0	3/4	1/8	1/8
2	0	0	1/2	1/2
3	0	0	0	1

Markov Decision Processes (MDP)

- Rather than just observing the Markov chain, we can take an action to take in each state.

- Given an (action, state) pair, we transition into the next state.
- Each state is associated with a cost (or value).

- The problem of choosing an action in each state is referred as an Markov Decision Process.

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Example: Machine Maintenance

State	Condition	Cost
0	Good as new	0
1	Minor deterioration	1000
2	Major deterioration	2000
3	Inoperable	6000

Decision	Action	Relevant States
1	Do nothing	0, 1, 2
2	Overhaul (return the system to state 1)	2
3	Replace (return the system to state 0)	1, 2, 3

Goal: find a maintenance plan that minimizes total expected cost

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AM 121: Learning Outcomes

- Be able to transform a real world problem into a mathematical programming model
- Master the techniques used to solve the mathematical programming model
- Be able to solve the problem using commercial software packages, i.e. CPLEX/AMPL
- Feel empowered!

- *Come see me during my office hours if you have questions about the course. Assignment zero posted!*

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