AM 121: Intro to Optimization Models and Methods

EO 2: Paired Kidney Donation

David C. Parkes

Matching Problems

- Two-sided matching
- One-sided matching with strict preferences
- One-sided matching

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I: Two-sided Matching

- Students to (public) schools; Medical interns to residencies; TAs to professors

- Agents in two sets; agent in one set has **strict** preferences over agents in other set
- A **matching**: each agent assigned to at most one agent on other side

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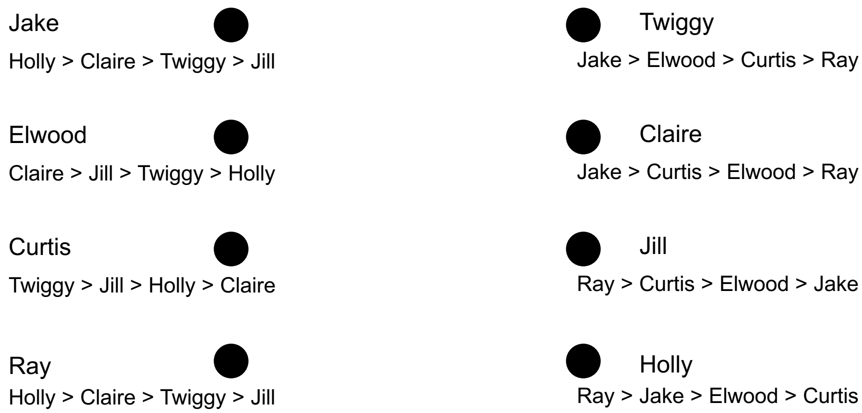
Example (unstable!)

the boys	and girls
Jake ♥ Holly > Claire > Twiggy > Jill	Twiggy Jake > Elwood > Curtis > Ray
Elwood Claire > Jill > Twiggy > Holly	Claire ♥ Jake > Curtis > Elwood > Ray
Curtis Twiggy > Jill > Holly > Claire	Jill Ray > Curtis > Elwood > Jake
Ray Holly > Claire > Twiggy > Jill	Holly Ray > Jake > Elwood > Curtis

(Immorlica/Hartline)

Boy proposing

the boys and girls



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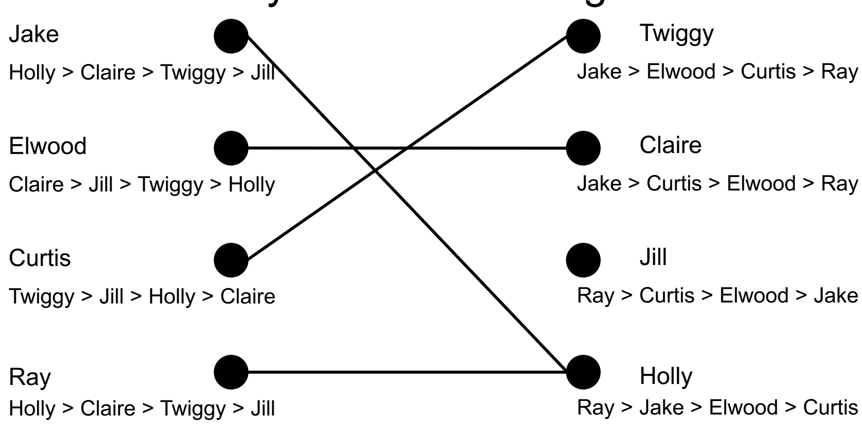

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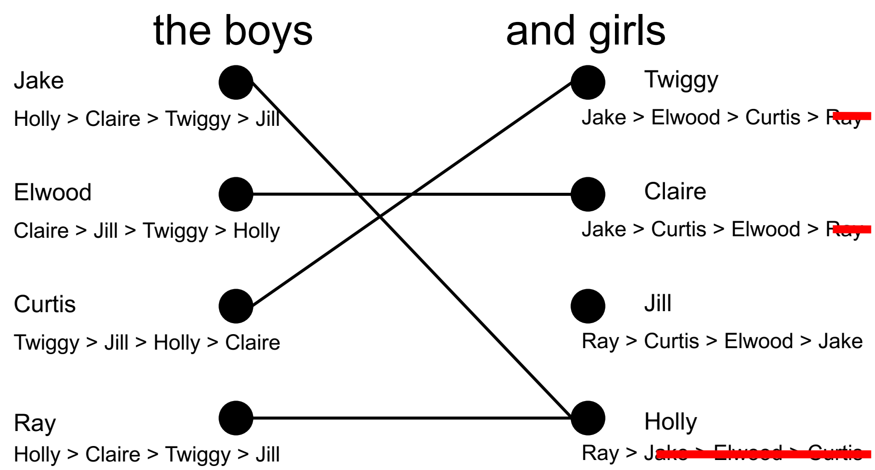

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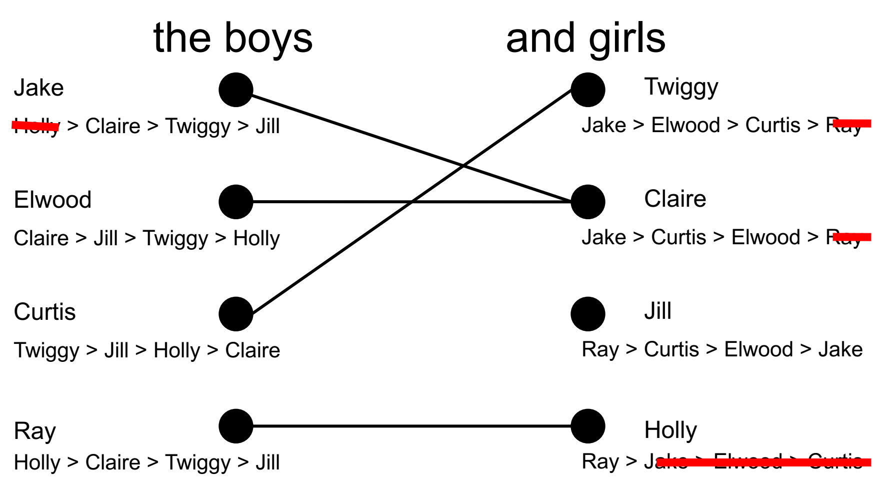

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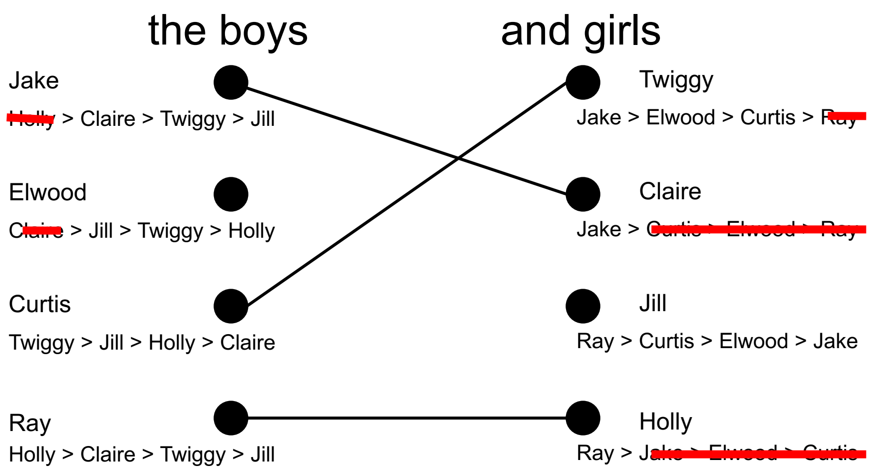

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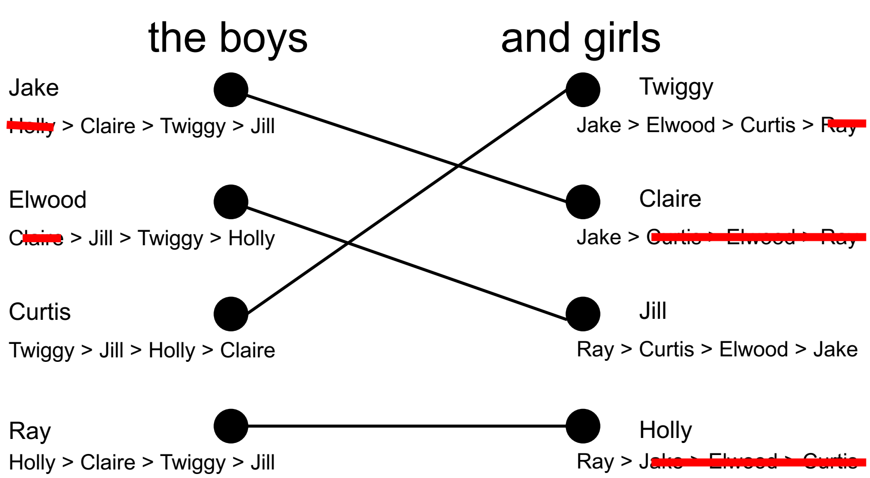
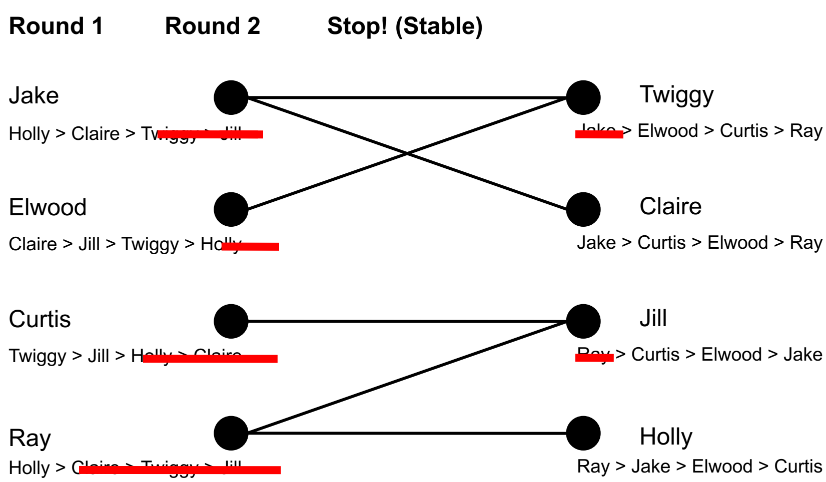


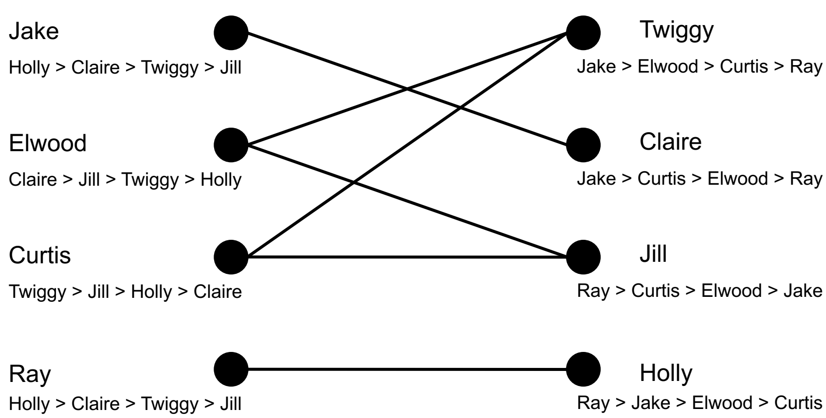
(Immorlica/Hartline)

Girl-Proposing Deferred Acceptance



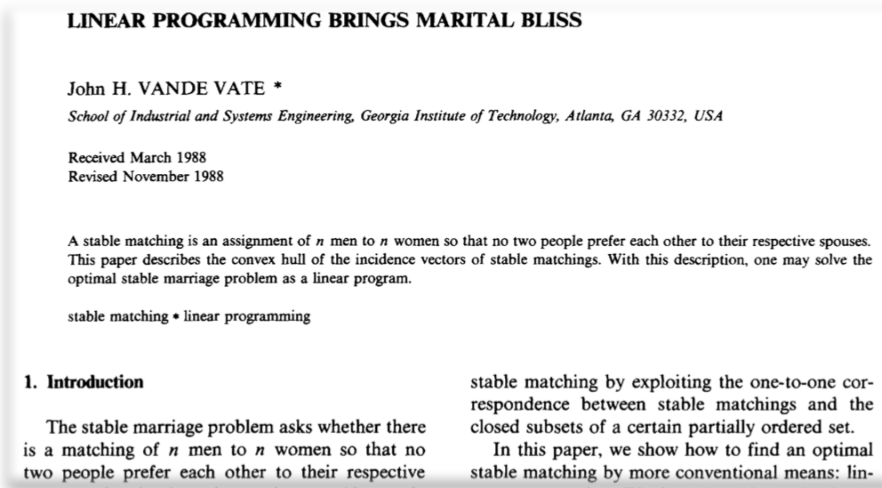
(Immorlica/Hartline)

Not unique!



(Immorlica/Hartline)

Revenge of the LP!



LINEAR PROGRAMMING BRINGS MARITAL BLISS

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Revised November 1988

A stable matching is an assignment of *n* men to *n* women so that no two people prefer each other to their respective spouses. This paper describes the convex hull of the incidence vectors of stable matchings. With this description, one may solve the optimal stable marriage problem as a linear program.

stable matching • linear programming

1. Introduction

The stable marriage problem asks whether there is a matching of *n* men to *n* women so that no two people prefer each other to their respective

stable matching by exploiting the one-to-one correspondence between stable matchings and the closed subsets of a certain partially ordered set.

In this paper, we show how to find an optimal stable matching by more conventional means: lin-

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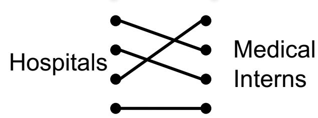
Real-world Matching Markets

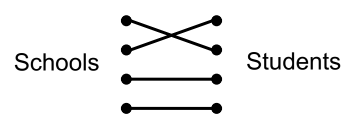
National Residency Matching Program (NRMP).



- Adoption of student-proposing NMRP in 1998
- Easier for students

- Practical concern: couples with preferences on pairs of positions

School Choice (Boston and New York).



- “Boston mechanism” was not stable or truthful
- Fix: adopt student-proposing DA

Real-world Matching Markets

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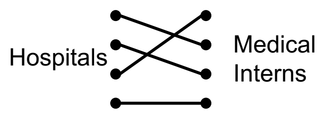

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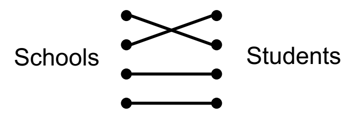

• "Boston mechanism" was not stable or truthful
• Fix: adopt student-proposing DA

• Easier, more fair, and allow for policy advice

• Practical concern: priorities for schools (siblings, walk zones)

One-sided Matching (strict prefs)

- Each agent "owns" an item (dorm rooms?)
- Strict preferences
- Solutions:
  - Random serial dictatorship (RSD)
  - Top trading cycle algorithm

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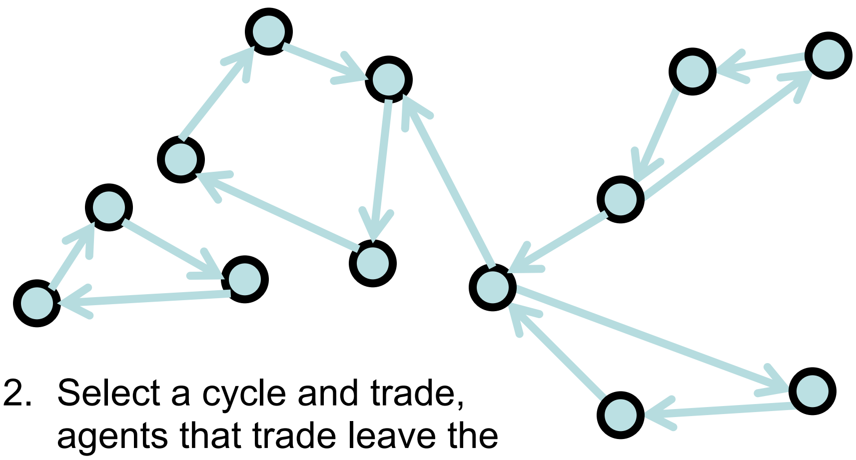
Top trading cycles



2. Select a cycle and trade, agents that trade leave the market.

(Hartline/Immorlica)

One-Sided Matching (0/1 prefs)

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Kidney disease

- Kidney failure serious medical problem
- Preferred treatment: kidney transplant
  - Cadaver kidneys
  - Donation from live patient
  - Must be blood- and tissue-type compatible

As of early 2016:

121,274 people

waiting for a kidney transplant in the US.

http://optn.transplant.hrsa.gov

In 2008,

10,526 patients
received cadaver kidneys.

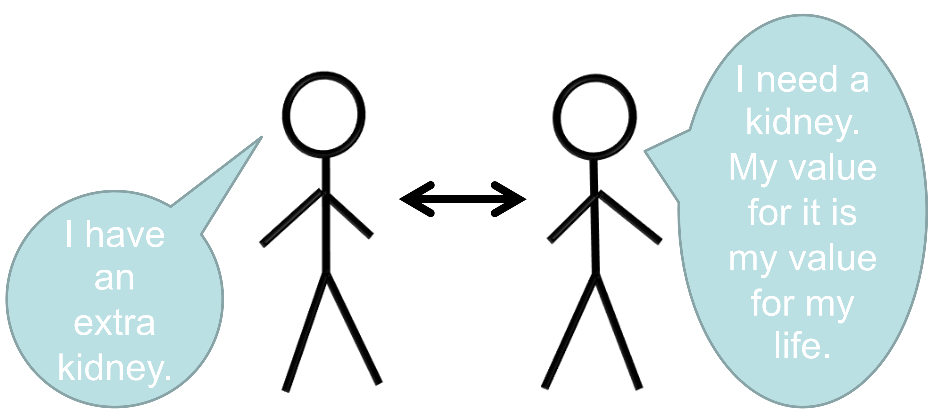
4,857 patients
received live donor kidneys.

5,920 patients died
or became too sick for a transplant.

(Hartline/Immorlica)

Econ 101

- Trade for $$s



(Hartline/Immorlica)

Legality

Section 301 of the National Organ Transplant Act, “Prohibition of organ purchases” imposes criminal penalties on any person who

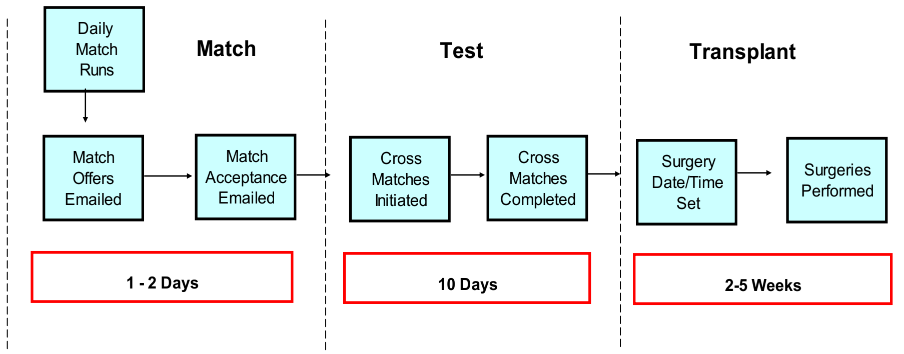
“knowingly acquire[s], receive[s], or otherwise transfer[s] any human organ for valuable consideration for use in human transplantation”

(Hartline/Immorlica)

Kidney exchanges (APD, UNOS, NKR)

1. Pairs register in database.
2. Form a graph, representing possible compatibilities
3. Feasible matchings found
4. Additional medical tests (“cross match”)
5. Transplants performed

Match Offer Process

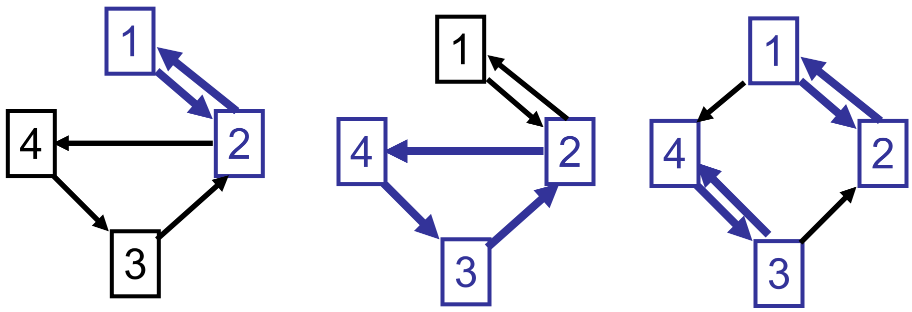


KX via top-trading cycles?

- No: 0/1 rather than strict preferences
- Limits on cycle lengths

Kidney-Paired Donation

- Find vertex-disjoint cycles of length <= *k* that cover as many vertices as possible

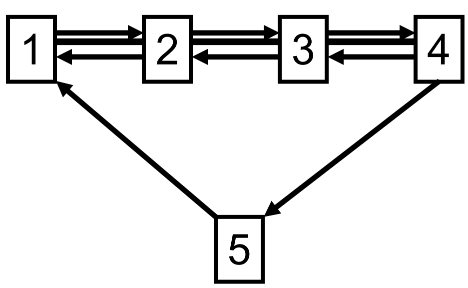


k=2 k=3 k=3

(Conitzer)

Special case: k=2

- If edges go in both directions, replace by undirected edge
- Remove other edges



- Find *max cardinality matching* (max #edges, every vertex incident on at most one edge)
- Edmond's algorithm (poly time.)

(Conitzer)

Complexity

- k = 2: in **P** by Edmonds alg
- k = ∞ in **P**

  (via a reduction to a maximum weight perfect matching problem)

(Conitzer)

K=infinity: Edge formulation

$$\max_y \sum_{(i,j)\in E} y_{ij}$$

$$\text{s.t.} \quad \sum_j y_{ij} \leq 1, \qquad \forall i$$

$$\sum_j y_{ij} = \sum_i y_{ji}, \qquad \forall i$$

$$y_{ij} \geq 0$$

Optimal solutions non-fractional!

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k > 2 (not not infinite)

Complexity

- k = 2: in **P** by Edmonds alg
- k = ∞ in **P**
- But k = 3, 4, 5, …: **NP**-hard!

(Conitzer)

Cycle formulation (IP)

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Doing better

- Cycle size limited because of logistical and ethical concerns

- What can be done?

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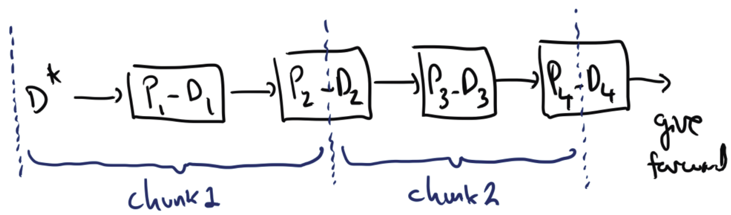
Altruistic Non-simultaneous Donor Chains

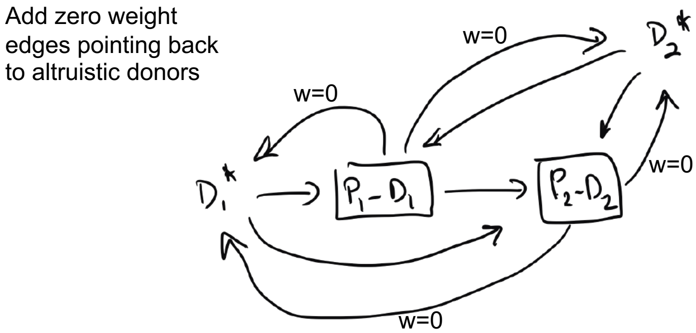
Altruistic Non-simultaneous Donor Chains

- "Good Samaritan" donors. Enable (long) chains



- New computational challenge. Can reduce "cycles and chains" to "cycles" via zero weight back edges. Allow longer cycles, but ONLY if they involve an altruistic donor.

New clearing problem



- Can only solve 100 pairs with k<= 20
- Active research into this problem
  - Note: cannot use unrestricted length cycles, because may not include an altruistic donor

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National Kidney Register

Completed 500+ transplants
Entirely "altruistic non-simultaneous chains"

- 60 person, 30 transplant chain (8/11- 12/11)

http://www.nytimes.com/interactive/2012/02/18/health/record-chain-of-kidney-transplants.html?ref=health

Extensions

- Dynamics: deciding how to batch the clearing decision (more matches vs fairness
- Incentive considerations: How to get hospitals to share lists in order to promote better matching?

- Panos Toulis, David C. Parkes: Design and analysis of multi-hospital kidney exchange mechanisms using random graphs. *Games and Economic Behavior* **91**: 360-382 (2015)

Other applications

- Peerflix (DVDs)
- Read It Swap It (books)
- Intervac (holiday houses)
- National odd shoe exchange

(Conitzer)

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