Real-World Applications of Stochastic Optimization: Part III
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The Inputs
The Inputs

Road network

Legend

- Evacuated node
- Transit node
- Safe node

Time at which the road is flooded

The Inputs

Road network

Legend

- Evacuated node
- Transit node
- Safe node

Time at which the road is flooded
The Inputs

Evacuation graph

(travel time, capacity, flooded time)

(demand, flooded time)
Scheduling Evacuations

Time-expanded evacuation graph

Going from 2 to B takes 1 hour
At most 10 vehicles per hour

Node 2 is flooded at 11:00

Going from 2 to B takes 1 hour
At most 10 vehicles per hour
Scheduling Evacuations in Practice

- Decisions
  - when to evacuate
  - where to evacuate
  - how to evacuate
  - when to use contraflows

- Objectives
  - Maximize the number of evacuated people
  - Delay the evacuation as much as possible

- Field constraints
  - Same route for all evacuees of a node
  - Always possible to partition the residential area further
Scheduling Evacuations

- Multi-commodity flows over time

\[
\max \Phi = \sum_{e \in \delta^+(v_s)} \sum_{k \in E_0} \varphi^k_e \\
\sum_{e_0 \in \delta^+(i)} x_{e_0}^k \leq 1 \quad \forall k \in E_0, i \in E_0 \cup T_0 \\
\sum_{e_0 \in \delta^-(d)} x_{e_0}^k \leq 1 \quad \forall k \in E_0, d \in S_0 \cup T_0 \\
\sum_{e_0 \in \delta^-(i)} x_{e_0}^k - \sum_{e_0 \in \delta^+(i)} x_{e_0}^k = 0 \quad \forall k \in E_0, i \in T_0 \\
\sum_{e \in \delta^-(i)} \varphi^k_e - \sum_{e \in \delta^+(i)} \varphi^k_e = 0 \quad \forall i \in N \setminus \{v_s, v_t\}, k \in E_0 \\
\sum_{k \in E_0} \varphi^k_e \leq u_e \quad \forall e \in A \\
\varphi^k_e \leq u_e \cdot x_{e_0}^k \quad \forall e \in A \quad \forall e_0 \in A_0, k \in E_0 \\
\varphi^k_e \geq 0 \quad \forall e \in A, k \in E_0 \\
x^k_e \in \{0, 1\} \quad \forall e \in A, k \in E_0
\]
Multi-Commodity Flows

- Multi-commodity flows over time
  - Scalability

185 nodes
458 edges

10h horizon
5min steps

?
The Path Generation Approach

Start

Generate initial paths

Solve the master scheduling problem

Identify critical nodes C

C=∅?

Generate new paths for nodes in

Add conflicting nodes to C

End
The Path Generation Approach

Start

Generate initial paths

Solve the master scheduling problem

Identify critical nodes $C$

$C = \emptyset$?

NO

Generate new paths for nodes in

YES

Add conflicting nodes to $C$

End
The Path Generation Approach

Start

Generate initial paths

Solve the master scheduling problem

Identify critical nodes \( C \)

\( C = \emptyset \) ?

NO

Generate new paths for nodes in

Add conflicting nodes to \( C \)

YES

End
Critical Nodes

Goal: generate paths that avoid this saturated edge shared by B and C.
The Path Generation Approach

1. Start
2. Generate initial paths
3. Solve the master scheduling problem
4. Identify critical nodes C
   - If C = ∅: Add conflicting nodes to C
   - Generate new paths for nodes in C

End
Scheduling Evacuations
Results

Total number of vehicles

Stochastic optimization

Simulation: evacuees go to the closest safe zone

61% cannot be

42,000 persons trapped in a flooded area
Forks in the Road

- When you arrive at a fork, take it
  – Yogi Berra

Fig. 3. I-10 Contraflow loading point (Kenner, Louisiana, September 14, 2004)
Convergent Flows

- Design an evacuation plan that has no fork
  - tree/forest structure structure
Convergent Plans

- MIP Model

\[
\begin{align*}
\max & \quad \sum_{e_t \in \delta^+ (v_s)} \varphi_{e_t} \\
\text{s.t.} & \quad \sum_{e_t \in \delta^- (i)} \varphi_{e_t} - \sum_{e_t \in \delta^+ (i)} \varphi_{e_t} = 0 \quad \forall i \in \mathcal{N}^x \quad (2) \\
& \quad \sum_{e \in \delta^+ (i)} x_e \leq 1 \quad \forall i \in \mathcal{N} \quad (3) \\
& \quad \varphi_{e_t} \leq x_e u_{e_t} \quad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (4) \\
& \quad \varphi_{e_t} \geq 0 \quad \forall e_t \in \mathcal{A}^x \quad (5) \\
& \quad x_e \in \{0, 1\} \quad \forall e \in \mathcal{A} \quad (6)
\end{align*}
\]
Tree Design Problem

\[
\begin{align*}
\max \quad & \sum_{e \in \delta^+(v_s)} \psi_e \\
\text{s.t.} \quad & \sum_{e \in \delta^-(i)} \psi_e - \sum_{e \in \delta^+(i)} \psi_e = 0 \quad \forall i \in \mathcal{N} \\
& \sum_{e \in \delta^+(i)} y_e \leq 1 \quad \forall i \in \mathcal{N} \\
& \psi_e \leq y_e \sum_{t \in \mathcal{H}} u_{et} \quad \forall e \in \mathcal{A} \\
& \psi_e \geq 0 \quad \forall e \in \mathcal{A} \\
& y_e \in \{0, 1\} \quad \forall e \in \mathcal{A}
\end{align*}
\]
Flow Scheduling

\[
\max \sum_{p \in \Omega} \sum_{t \in \mathcal{H}_p} \chi_p^t \\
\text{s.t. } \sum_{t \in \mathcal{H}_p} \chi_{p_i}^t \leq d_i \quad \forall i \in \mathcal{E} \\
\sum_{p \in \omega(e)} \chi_{p}^{t-\tau^e_p} \leq u_{e_t} \quad \forall e \in \mathcal{A}, t \in \mathcal{H} \\
\chi_p^t \geq 0 \quad \forall p \in \Omega, t \in \mathcal{H}_p
\]
## Convergent Plans: Results

<table>
<thead>
<tr>
<th>Instance</th>
<th>TDFS</th>
<th>MIP 24h</th>
</tr>
</thead>
<tbody>
<tr>
<td>HN80</td>
<td>33.4</td>
<td>100%</td>
</tr>
<tr>
<td>HN80-I1.1</td>
<td>1.3</td>
<td>100%</td>
</tr>
<tr>
<td>HN80-I1.2</td>
<td>1.0</td>
<td>100%</td>
</tr>
<tr>
<td>HN80-I1.4</td>
<td>6.4</td>
<td>100%</td>
</tr>
<tr>
<td>HN80-I1.7</td>
<td>47.6</td>
<td>100%</td>
</tr>
<tr>
<td>HN80-I2.0</td>
<td>4.1</td>
<td>95.5%</td>
</tr>
<tr>
<td>HN80-I2.5</td>
<td>6.6</td>
<td>80.7%</td>
</tr>
<tr>
<td>HN80-I3.0</td>
<td>1.5</td>
<td>67.7%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>14.1</strong></td>
<td><strong>93.0%</strong></td>
</tr>
</tbody>
</table>

Table 2: Primal Bounds for the HN80-I Instances.