

AM121/ES121: Useful L^AT_EX code snippets

You should find these code snippets very useful for your assignment and extreme optimization write-ups in L^AT_EX. You are free to copy and past the code in this document into your files and modify it to produce your solutions to the tasks.

The snippets in this document are relatively advanced and the last few are specific for linear programming. If you are a L^AT_EX beginner, you should use the L^AT_EX cheat sheets under Resources. You will find more comprehensive explanations at and examples at websites such as “WikiBooks L^AT_EX guides” at *en.wikibooks.org/wiki/Latex*. You can also get quick answers to your L^AT_EX questions through search engines.

We recommend downloading a user-friendly L^AT_EX program such as “TexShop” (on Mac) or “WinEdt” (on PC) to code and compile. TexShop can be downloaded as a package for free online, and it has many snippets of code in its Macros list. WinEdt can be downloaded from FAS IT downloads. Other T_EX software exists for free download online, and can be searched and found quite easily.

If you are having trouble downloading or coding in your L^AT_EX software, first check for answers to your question online, in the cheat sheets under Resources, or in this document. For example, a useful website for finding the L^AT_EX code for special symbols is *detexify.kirelabs.org/classify.html*. If you cannot find an answer to your question, please do not hesitate to post on the message board or ask your TF.

Contents

1	Graphics	2
2	Equation formatting	2
3	Aligned equations	4
4	Tables	5
5	Matrices and arrays	6
6	Basic linear program format	7
7	Full model layout and definitions	8

Snippet 1 Graphics

To insert a graphic into your L^AT_EX write-up, use the following code. Substitute the name of your graphic into “examplefilename” (try with the file extension if it does not compile). You can adjust the width of the graphic by varying the “width” parameter.

```
\begin{center}
  \includegraphics[width=150mm]{examplefilename}
\end{center}
```

To use a caption for your figure and to label it so you can easily refer to it in your write-up text (with `\ref{examplelabel}`), use the figure environment.

```
\begin{figure}[h!]
  \centering
  \includegraphics[width=150mm]{examplefilename}
  \caption{examplecaption}
  \label{examplelabel}
\end{figure}
```

For additional information, see a L^AT_EX guide on graphics and figure environments, such as the following at WikiBooks L^AT_EX guides:

1. en.wikibooks.org/wiki/LaTeX/Importing_Graphics
2. en.wikibooks.org/wiki/LaTeX/Floats,_Figures_and_Captions

Snippet 2 Equation formatting

Here is an example of an inequality concept that we will use in the course.

The Chvátal-Gomory cut for row u is:

$$x_{B_u} + \sum_{j \in NB} [\bar{a}_{uj}] x_j \leq [\bar{b}_u]$$

Rewriting this inequality by eliminating x_{B_u} gives:

$$\sum_{j \in NB} (\bar{a}_{uj} - [\bar{a}_{uj}]) x_j \geq \bar{b}_u - [\bar{b}_u]$$

The code for producing such inequalities is quite simple and is displayed below. Note the use of double `$$` to align the equation in the center of the paper. A single `$` on either side of the equation would not center the equation on a new line, it would print the equation in the text of a paragraph.

The Chvátal-Gomory cut for row u is:

$$x_{B_u} + \sum_{j \in NB} \lfloor \overline{a}_{uj} \rceil x_j \leq \lfloor \overline{b}_u \rfloor$$

Rewriting this inequality by eliminating x_{B_u} gives:

$$\sum_{j \in NB} (\overline{a}_{uj} - \lfloor \overline{a}_{uj} \rfloor) x_j \geq \overline{b}_u - \lfloor \overline{b}_u \rfloor$$

Here is another example of a notation-heavy inequality that would not fit on a single line without several commands that make it compact. First, the sums in this equation are indexed over long set names. So we use the `\substack{}` command to split these set names into two (or more) lines beneath the sums. Second, the “for all” (\forall) statement of indexes over which the constraint is defined will not fit on the same line as the general constraint inequality. To fix this and to enable us to label the whole set of constraints with the equation number (1), we use the `split` environment. See the code below the equation output below.

$$\sum_{\substack{q \in PROD: \\ q \leq p}} \sum_{\substack{i \in ORIG \cup TRAN: \\ (i,j) \in ARCS}} Flow_{ij}^q + Ddev_j^p \geq \sum_{\substack{q \in PROD: \\ q \leq p}} dema_j^q \quad (1)$$

$\forall p \in PROD, j \in DEST$

```

\setcounter{equation}{0}
\begin{align}
\begin{split}
& \sum_{\substack{q \in PROD: \\ q \leq p}} \\
& \sum_{\substack{i \in ORIG \cup TRAN: \\ (i,j) \in ARCS}} Flow_{ij}^q + Ddev_j^p \\
& \geq \sum_{\substack{q \in PROD: \\ q \leq p}} dema_j^q \\
& \forall p \in PROD, j \in DEST
\end{split}
\end{split}
\end{align}

```

As an additional reference, use textit.wikibooks.org/wiki/LaTeX/Mathematics or search online for answers.

Snippet 3 Aligned equations

We can create the following output with equations that are vertically aligned along the inequality and set operator signs. The L^AT_EX environment responsible for this orderly alignment of equations is the `align` environment (`\begin{align}... \end{align}`):

Let there be m binary variables y_i , $i = 1, \dots, m$. Since all $a'_i x \geq f$, set some big-M to $M = \min_i \{b_i - f\}$. Now we can write:

$$a'_i x \geq b_i - (1 - y_i)M \quad \forall i \quad (1)$$

$$\sum_{i=1}^m y_i \geq k \quad (2)$$

$$y_i \in \{0, 1\} \quad \forall i \quad (3)$$

Equation (2) forces at least k of the $a'_i x \geq b_i$ equations as part of (1) to hold.

Note that the text above is indented using the `quote` environment (`\begin{quote}... \end{quote}`).

The output is produced by the following L^AT_EX code:

Let there be m binary variables y_i , $i=1, \dots, m$.

Since all $a_i^{\prime} x \geq f$, set some big-M to $M = \min_i \{b_i - f\}$. Now we can write:

```
\setcounter{equation}{0}
\begin{align}
  a_i^{\prime} x & \geq b_i - (1 - y_i)M & \forall i \\
  \sum_{i=1}^m y_i & \geq k \\
  y_i & \in \{0, 1\} & \forall i
\end{align}
```

Equation (2) forces at least k of the

$a_i^{\prime} x \geq b_i$ equations as part of (1) to hold.

Note that in the `align` environment, the ampersands (&) are used to fix vertical locations in series on a line of text. A line is ended with the double backslash (`\`). To avoid the automatic numbering of lines or equations, as in the example output above, use `align*` in place of `align`.

Snippet 4 Tables

Using the `tabular` environment, we can produce the following table:

col 1	col 2	col 3
row 1	5	$\sum_{i=1}^9 x_i$
row 2	$x + x^2$	blue

```
\begin{center}
\begin{tabular}{r|cc}
\hline
col 1 & col 2 & col 3 \\ \hline
row 1 & 5 &  $\sum_{i=1}^9 x_i$  \\
row 2 &  $x+x^2$  & blue \\
\hline
\end{tabular}
\end{center}
```

The `{r|cc}` following the beginning of the `tabular` environment may be modified to adjust the number of columns, the alignment of text in the cells of these columns (l is left, c is center, r is right), and the lines drawn between columns (a vertical bar | creates a line). The `\hline` command, which follows a new line command `\\`, draws a horizontal line between rows. Here is another example of a useful table for this class (a “tableau” in optimization jargon):

-10	δ	-2	0	0	0
4	-1	η	1	0	0
1	α	-4	0	1	0
β	γ	3*	0	0	1

It is produced with the following code:

```
\begin{center}
\begin{tabular}{|c|ccccc|}
\hline
-10 &  $\delta$  & -2 & 0 & 0 & 0 \\ \hline
4 & -1 &  $\eta$  & 1 & 0 & 0 \\
1 &  $\alpha$  & -4 & 0 & 1 & 0 \\
 $\beta$  &  $\gamma$  & 3* & 0 & 0 & 1 \\
\hline
\end{tabular}
\end{center}
```

Matrices, tables, and arrays are all quite similar environments, and learning how to use them well will take some trial and error. TeXShop for Mac has good table macros (code snippets and built-in functions) under the “Macros” list. For further reference and more advanced commands for the `tabular` environment, see online resources such as WikiBooks L^AT_EX guide, at en.wikibooks.org/wiki/LaTeX/Tables.

Snippet 5 Matrices and arrays

Below is a simple matrix:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

It is formatted in a simple way using the `pmatrix` environment, as follows:

```

 $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ 

```

Here is a tableau in matrix form, with the variable names explicitly labeled:

$$\begin{bmatrix} \textit{vars} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \textit{RHS} \\ z & 0.1 & 0.3 & 0 & 0.2 & 0 & 0 & 23.1 \\ x_6 & 1.3 & -0.3 & 0 & -1.0 & 0 & 1 & 5.3 \\ x_3 & 0 & 1.1 & 1 & 0.4 & 0 & 0 & 1.6 \\ x_5 & -0.8 & -0.2 & 0 & -0.5 & 1 & 0 & 3.7 \end{bmatrix}$$

It is formatted with the `array` environment, as follows:

```

 $\left[ \begin{array}{rrrrrrrr} \textit{vars} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \textit{RHS} \\ z & 0.1 & 0.3 & 0 & 0.2 & 0 & 0 & 23.1 \\ x_6 & 1.3 & -0.3 & 0 & -1.0 & 0 & 1 & 5.3 \\ x_3 & 0 & 1.1 & 1 & 0.4 & 0 & 0 & 1.6 \\ x_5 & -0.8 & -0.2 & 0 & -0.5 & 1 & 0 & 3.7 \end{array} \right]$ 

```

For more advanced matrix and array commands, see online resources such the math section of the WikiBooks \LaTeX guide, at en.wikibooks.org/wiki/LaTeX/Mathematics.

Snippet 6 Basic linear program format

The following basic LP model in standard form:

$$\begin{aligned} \min \quad & c'x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

Is formatted with the following code:

```
\begin{align*}
\text{\text{min}} \quad & \text{\text{min}} \quad \text{\text{min}} \\
c^{\prime} x & \quad \quad \quad \\
\text{\text{s.t.}} \quad & \text{\text{s.t.}} \quad \text{\text{s.t.}} \\
Ax = b & \quad \quad \quad \\
x \geq 0 & \quad \quad \quad \\
\end{align*}
```

To write a simple LP primal problem alongside its dual, as below:

Consider this primal LP and its dual, for $M_1 \cup M_2 = M$ and $N_1 \cup N_2 = N$:

$$\begin{array}{ll} \text{minimize} & c'x \\ \text{subject to} & a'_i x = b_i, \quad i \in M_1 \\ & a'_i x \geq b_i, \quad i \in M_2 \\ & x_j \geq 0, \quad j \in N_1 \\ & x_j \text{ free}, \quad j \in N_2 \end{array} \quad \begin{array}{ll} \text{maximize} & p'b \\ \text{subject to} & p_i \text{ free}, \quad i \in M_1 \\ & p_i \geq 0, \quad i \in M_2 \\ & p'A_j \leq c_j, \quad j \in N_1 \\ & p'A_j = c_j, \quad j \in N_2 \end{array}$$

Use the following code:

Consider this primal LP and its dual,
for $M_1 \cup M_2 = M$ and $N_1 \subset N$:

```
\begin{align*}
\text{\text{minimize}} \quad & \text{\text{minimize}} \quad c^{\prime} x \\
& \text{\text{maximize}} \quad p^{\prime} b \\
\text{\text{subject to}} \quad & \text{\text{subject to}} \quad a^{\prime}_i x = b_i, \quad i \in M_1 \\
& \text{\text{subject to}} \quad p_i \text{\text{ free}}, \quad i \in M_1 \\
& a^{\prime}_i x \geq b_i, \quad i \in M_2 \\
& p_i \geq 0, \quad i \in M_2 \\
& x_j \geq 0, \quad j \in N_1 \\
& p^{\prime} A_j \leq c_j, \quad j \in N_1 \\
& x_j \text{\text{ free}}, \quad j \in N_2 \\
& p^{\prime} A_j = c_j, \quad j \in N_2 \\
\end{align*}
```

Snippet 7 Full model layout and definitions

The following example layout for a full mathematical model of an LP is below. When you are not given sets and parameters and variables explicitly, you are expected to define them clearly and format them as in this example. See the “mathematical programming style guide” in the Resources tab for further general information about laying out a full mathematical model.

The model presented below is the “multicommodity flow model”, a famous general network linear program for finding the optimal constrained flows of multiple commodities on a network. The L^AT_EX code and a detailed description of the model is presented after.

Sets

- N set of nodes
- A set of arcs
- K set of commodities

Parameters

- u_{ij} $(i, j) \in A$ total flow capacity of arc (i, j)
- u_{ij}^k $k \in K, (i, j) \in A$ flow capacity for commodity k on arc (i, j)
- c_{ij}^k $k \in K, (i, j) \in A$ unit flow cost of commodity k on arc (i, j)
- b_i^k $k \in K, i \in N$ supply/demand for commodity k at node i

Variables

- x_{ij}^k $k \in K, (i, j) \in A$ flow of commodity k on arc (i, j)

Objective

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \quad (1)$$

Constraints

$$\sum_{j:(i,j) \in A} x_{ij}^k - \sum_{j:(j,i) \in A} x_{ji}^k = b_i^k \quad \forall k \in K, i \in N \quad (2)$$

$$\sum_{k \in K} x_{ij}^k \leq u_{ij} \quad \forall (i, j) \in A \quad (3)$$

$$0 \leq x_{ij}^k \leq u_{ij}^k \quad \forall k \in K, (i, j) \in A \quad (4)$$

The code for this model set up follows:

```

Sets\\
\begin{tabular}{ll}
  N$ & set of nodes \\
  A$ & set of arcs \\
  K$ & set of commodities
\end{tabular}

Parameters\\
\begin{tabular}{llll}
  u_{ij}$ & $(i,j) \in A$ & total flow capacity of arc $(i,j)$ \\
  u_{ij}^k$ & $k \in K, (i,j) \in A$ & flow capacity for commodity $k$ on arc $(i,j)$ \\
  c^k_{ij}$ & $k \in K, (i,j) \in A$ & unit flow cost of commodity $k$ on arc $(i,j)$ \\
  b_i^k$ & $k \in K, i \in N$ & supply/demand for commodity $k$ at node $i$
\end{tabular}

Variables\\
\begin{tabular}{llll}
  x^k_{ij}$ & $k \in K, (i,j) \in A$ & flow of commodity $k$ on arc $(i,j)$
\end{tabular}

\setcounter{equation}{0}
Objective
\begin{align}
  \min \quad & \sum_{k \in K} \sum_{(i,j) \in A} c^k_{ij} x^k_{ij}
\end{align}
Constraints
\begin{align}
  \sum_{j:(i,j) \in A} x^k_{ij} - \sum_{j:(j,i) \in A} x^k_{ji} & = b_i^k \quad \&\& \text{forall } k \in K, i \in N \\
  \sum_{k \in K} x^k_{ij} \leq u_{ij} \quad \&\& \text{forall } (i,j) \in A \\
  0 \leq x^k_{ij} \leq u_{ij}^k \quad \&\& \text{forall } k \in K, (i,j) \in A
\end{align}

```

A more detailed description of this multicommodity flow model formulation is given below, but note that we do not expect such an explanation unless we explicitly ask for it. In the extreme optimizations, you should give rigorous descriptions of your models like the following.

$G = \langle N, A \rangle$ is the directed network defined by a set N of n nodes and a set A of m directed arcs, and K is the set of $|K|$ commodities that flow on G . The cost of a unit of commodity $k \in K$ flowing on arc $(i, j) \in A$ is c_{ij}^k . Arc (i, j) has a total flow capacity for all commodities of u_{ij} and a per-commodity flow capacity of u_{ij}^k units of commodity k . Each node $i \in N$ is associated with a supply/demand for commodity k of b_i^k . The decision variables are the nonnegative arc flows x_{ij}^k , representing how many units of commodity k flow on arc (i, j) .

The mass balance constraints in (2) state that for a commodity k , the outflow minus the inflow at a node must equal the supply/demand of that node.

The objective function in (1) minimizes the sum, over all commodities and all arcs in the network, of the flow costs $c_{ij}^k x_{ij}^k$.

The bundle constraints in (3) tie together the commodities by restricting the total flow of all commodities on arc (i, j) to an arc capacity of u_{ij} .

The individual flow bounds in (4) model physical capacities or restrictions placed on the operating ranges of flow for a specific commodity. Here, the lower bounds are assumed to be zero, which is appropriate for most applications.