

# Problem Set 0

## Getting Ready for the Course

AM121/ES121 — Fall 2019

Due 5:00 PM, Monday, September 9, 2019

Welcome to AM/ES 121! This assignment will help you get ready for the course. It contains a brief review of some linear algebra concepts, which the materials of this course will build upon, and instructions to help you set up for the course. You will not be graded for this assignment and there's nothing to turn in.

### 1 Refresh your linear algebra knowledge

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#### Task 1

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##### Linear algebra review.

When learning the Simplex method for solving linear programs (in about two weeks), you will need to have a solid understanding of the definition and properties of a *basis* for a vector space as well as basic matrix operations. The following are some problems to help you check your understanding on these concepts. Go over these problems and make sure that you know how to answer them. If you have any doubts about your answers, it's a good time to visit your linear algebra book. We will not post solutions to these problems, but please feel free to ask questions on Piazza (see Task 3).

1. Consider the set  $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \right\}$ .

(a) Write the vector  $\vec{u} = \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix}$  as a linear combination of the vectors in  $S$ .

(b) Is  $S$  a linearly independent set?

(c) Does  $S$  span  $\mathbb{R}^3$ ?

(d) Is  $S$  a basis for  $\mathbb{R}^3$ ?

(e) Let  $W$  be a linearly independent set of three vectors in  $\mathbb{R}^3$ ? Is  $W$  a basis for  $\mathbb{R}^3$ ?

2. Let  $A$  be an  $m \times n$  matrix.

(a) Suppose  $m > n$ . Under what conditions, if any, do the columns of  $A$  form a basis for  $\mathbb{R}^m$ ?

(b) Suppose  $m < n$ . Under what conditions, if any, do the columns of  $A$  form a basis for  $\mathbb{R}^m$ ?

3. Let

$$A = \begin{pmatrix} 2 & 4 & -2 & 1 \\ -2 & 5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{pmatrix}.$$

- (a) Find a basis for  $\text{Col } A$ , the column space of  $A$ .
  - (b) Find a second basis for  $\text{Col } A$ .
  - (c) What is  $\text{rank } A$ ?
4. Let  $A$  be an  $3 \times 6$  matrix with columns  $\vec{a}_i, i = 1, \dots, 6$ , and  $\vec{x}$  be a column vector in  $\mathbb{R}^6$ . By expanding out the product  $A\vec{x}$ , show that

$$A\vec{x} = A_e\vec{x}_e + A_o\vec{x}_o$$

where  $A_e$  and  $A_o$  are matrices formed from the even and odd indexed columns of  $A$ , respectively. Similarly,  $\vec{x}_e$  and  $\vec{x}_o$  are vectors consisting of the even and odd indexed entries of  $\vec{x}$ .

5. Let  $A, B$ , and  $X$  be invertible  $n \times n$  matrices. Solve the matrix equation below for  $X$ , simplifying your answer as much as possible.

$$(A^{-1}X)^{-1} = (AB^{-1})(AB^2)$$

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**End Task 1**

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## 2 Setting up for AM121/ES121

Now let's get you set up for the course!

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**Task 2**

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**Know the course website.**

Visit the course website at <http://am121.seas.harvard.edu>. Check out the Resources page. In particular, read Course Logistics Overview. All course materials will be posted at this website.

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**End Task 2**

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**Task 3**

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**Start using Piazza.**

1. Go to the AM121/ES121 class page on Piazza (<http://piazza.com/harvard/fall2019/am121es121>) and post your first question or note. Write up something very short— share with the class an interesting Optimization article, or ask the course staff and other students some relevant question such as “Why did you decide to take AM121/ES121?”. Now, label your post with the relevant “tag”. Drag the “pset0” tag onto your question or note. Look at the other tags to see if there is another that also describes your post. For example if your question is about office hours, tag your post with “office\_hrs”. If you posted an interesting Optimization article, tag your post with “interesting”. You should always tag your posts.

Piazza should be the place that you ask the course staff questions related to course materials. The course staff will respond to your question or note next time one of us checks Piazza. Other students may also respond, unless you make your post “private” i.e. share it only with instructors.

2. Now you will make a comment on someone else's post in Piazza. Look through other students' posts, either by clicking on a post in the left sidebar, or by clicking a tag such as “pset0” at the top of the page to find a post that has used this tag. This is another chance to experiment! “Like” someone's post by clicking “Good question” (if it really is). Start a followup discussion and reply with your answer. Test out some L<sup>A</sup>T<sub>E</sub>X code by using the math environment symbols; for example, the code  $\$$(\sum_{i=1}^5 x_i) + y$$$ will produce  $(\sum_{i=1}^5 x_i) + y$  as output.$

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**End Task 3**

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Please also take note of the anonymous feedback form on the course website. Feel free to use this to provide any comments or suggestions to the course staff at any time.

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**Task 4**

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### **Setup AMPL on your computer.**

Follow the Piazza posts to set up AMPL and the corresponding optimization solvers on your computer. You will need to use AMPL for assignment 1.

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**End Task 4**

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## **3 Turning in your assignment**

All of the problem sets in AM121/ES121 will have a “Final task” that summarizes what you should submit to the course staff. Follow the directions below to submit your first assignment. **(In this case, there’s nothing to turn in!)**

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**Final Task 5**

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You must finish all tasks by 5:00 PM, Monday, September 9, 2019.

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**End Task 5**

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